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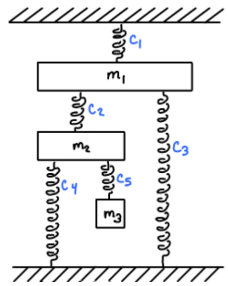
Assignment Spring_System_Equilibrium due 12/17/2021 at 02:07pm EET

Problem 1. **(1 point)** `METUNCC/Applied_Math/springs/spring-random_A_K.pg`

Consider the following spring system.



with spring constants $\mathbf{c} = \begin{bmatrix} 5 \\ 3 \\ 2 \\ 4 \\ 1 \end{bmatrix}$.

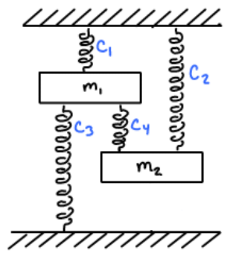
Assume down is the positive direction.

Write the elongation matrix $A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

Write the stiffness matrix $K = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

Problem 2. **(1 point)** `METUNCC/Applied_Math/springs/spring-random_force.pg`

Consider the following spring system.



with spring constants $\mathbf{c} = \begin{bmatrix} 2 \\ 2 \\ 5 \\ 2 \end{bmatrix}$.

Assume down is the positive direction.

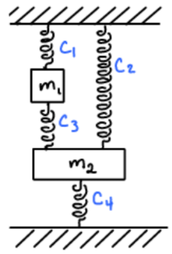
Write the stiffness matrix $K = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

- Compute the **external force which causes** the displacement $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

  Force = $\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

Problem 3. **(1 point)** `METUNCC/Applied_Math/springs/spring-random_displ.pg`

Consider the following spring system.



with spring constants $\mathbf{c} = \begin{bmatrix} 4 \\ 1 \\ 2 \\ 1 \end{bmatrix}$.

Assume down is the positive direction.

Write the stiffness matrix $K = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$

- Compute the displacements **caused by the external forces** $\mathbf{f} = \begin{bmatrix} 16 \\ -2 \end{bmatrix}$.

  Displacement = $\begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$

Problem 4. **(1 point)** `METUNCC/Applied_Math/springs/spring-balance_eq.pg`

An intersting thing happens when springs systems have no attachments to the outside. Consider the following free system.

with spring constants $\mathbf{c} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$.

Assume down is the positive direction.

Write the elongation matrix.

$$\mathrm{A} = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Free Displacements.

Because the system is unattached, it is possible to displace the masses without causing any internal force. We will find a (nonzero) displacement vector $\mathbf{u}$ so that

$$\mathrm{K}\mathbf{u} = \mathrm{A}^{\mathrm{T}}\mathrm{CA}\mathbf{u} = \mathbf{0}.$$

(1) Solve $\mathrm{A}^{\mathrm{T}}\mathbf{w} = \mathbf{0}$ (forward substitution).

$$\mathbf{w} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$$

(2) Solve $\mathrm{C}\mathbf{e} = \mathbf{w}$ (forward substitution).

$$\mathbf{e} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$$

(3) Solve $\mathrm{A}\mathbf{u} = \mathbf{e}$ (back substitution).

$$\mathbf{u} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} t$$

the same **u** *(which you just computed) in their null space, which corresponds to displacing the entire spring system all together. Since it doe*

Balanced Forces.
Just as there are certain displacements which cause no force; for this system, many external forces cannot be balanced by displacement. We can compute which forces can be balanced by investigating when there is a solution to

$$\mathrm{K}\mathbf{u} = \mathrm{A}^{\mathrm{T}}\mathrm{C}\mathrm{A}\mathbf{u} = \mathbf{f}.$$

(1) Write the LU decomposition for A^{T}

$$\mathrm{A}^{\mathrm{T}} = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix} \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$$

(2) Using variables for the components of **f**, divide by L (the left matrix above) using forward substitution.

$$\mathrm{L} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

(Write f1 *for* f_1*, and* f2 *for* f_2*, and* f3 *for* f_3*.)*

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$$

(3) Since the bottom row of U is all 0, there will only be a solution to $\mathrm{U}\mathbf{u} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ if

$\_\_\_ = 0$

(4) Did you really understand this? Write a nonzero force vector which is balanced by displacement.

$$\mathbf{f} = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$$

u apply this force to the spring system, the masses won't just move to a new equilibrium position... instead the whole spring system will fly av

Final Comments.
Note that both of the parts above (finding motions that did not cause force and forces that were not balanced by motion) relied only on the elongation matrix A rather than all of K. **We will see this again later when we look at collapse mechanisms for truss systems.**

- To find motions that did not cause internal force, we computed the **null space** of A.

- To find forces that could not be balanced by internal displacement (and thus would cause the whole spring system to move without any equilibrium), we computed the **column space** of A^T.

The **column space** of A^T is equal to the **row space** of A which is perpendicular to the **null space**. This is why your answers to the first and second parts looked so similar. (The motion vector in the first part is the normal vector for the plane of forces in the second part.)